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SUBJECT: Analysis of a Cylindrical pitot-static device for use in Air Flow Measurement

BY: Robert O. Brandt, Jr, PE

An analysis was done on a Cylindrical pitot-static device to determine its suitability for use as a flow measuring device. This analysis was carried out in four sections:

1. Fluid Velocity Measurement by Pitot Techniques- which describes a standard ellipsoidal pitot static device for comparison purposes
2. Pressure Distribution Around a Cylinder - mathematically investigates the pressure distribution around a cylinder in a flowing air stream
3. Using a Cylinder as a Pitot-uses the mathematical basis in **Section 2** to determine theoretically how the Cylinder Pitot should perform
4. Test of An Actual Example of a Cylindrical Pitot-Fechheimer Static Pitot-a test of an actual example of a multi-ported cylindrical pitot and conclusions

Conclusions: I would not recommend using the cylindrical pitot-static for any type of air flow measurement. The best design would be similar to the reference pitot described in **Section 1**. Following are some pertinent comments from the report and the sections found.

from 3.2.1 in the body of the report:
1. The use of the cylindrical "static" ports causes extreme sensitivity of the calibration factor to the angular location of the "static" ports in manufacturing. To maintain 1% repeat between probes, this angle would have to be maintained within 0.3° (See *Eq. 3.1*). This type of cylindrical "static" port is sometimes called "Fechheimer" static. **Additionally, slight build up on the surface of the "Fechheimer" static will cause severe calibration shifts due to the sensitivity of the static port.**

additionally, from 3.2.3
3. The extreme sensitivity to Yaw angle of the Cylindrical pitot-cylindrical static makes it almost impossible to align instrument with flow direction. Most pitot-static devices are usually no more than ±1% error for ±10° yaw (see *Graph 1.1*). The Cylindrical pitot - Cylindrical static has ~10x this error in the theoretical model (See Graph 3.2). Since the "Fechheimer static" port is so sensitive to Yaw, it would make a good device to determine flow direction. One would need only to connect the two "Fechheimer statics" to some type of differential meter and adjust for null, at which time the point on the cylinder between the two would point in the flow direction. Port separation less than ±30° would reduce the sensitivity while ±30° to ±40° would be about the same sensitivity.

and from the last paragraph of 4.4.1
........Based on the nominal diameter of 0.046 inches for the measured diameter Fstatic port, an error of ±0.001 inch would result in a ~15% pressure offset--very nearly what we are observing. This is a very difficult manufacturing issue, but even more important, an operational issue as the ports slightly plug.(See *Eq. 4.6*)

from 4.4.4
This theoretical analysis and actual measurement of a cylindrical pitot-static supports the conclusion in **Ref. 1**:Ower, E. & Pankhurst, R. C. The Measurement of Air Flow, 5th ed., Pergamon Press, pg, 38, last paragraph "the pitot cylinder cannot be used for accurate determinations of flowrates in pipes..." He does conclude at the end of the section on pg. 39 that "the convenience of the pitot cylinder for local exploration of the flow will favor its use for many purposes where high accuracy is not essential. ...provided that the static pressure is determined by some other device."

Author's note on the primary reference--The Measurement of Air Flow : while currently out of print and the 5th ed. has a 1977 date, it is considered the "Bible" for air flow measurement techniques. Used copies are still available over the Internet.

1. Fluid Velocity Measurement by Pitot Techniques

The measurement device of the free velocity stream must be stationary and some fixed surface port must be used. The solution to the free stream velocity requires that the $P_{Dynamic}$ must be determined. If every point also has static pressure, then the measurement of $P_{Dynamic}$ must be a differential measurement that leaves only the $P_{Dynamic}$ component.

Any measurement made in a free stream is a total pressure which obeys the following equation

$P_{Total} := P_{Dynamic} + P_{Static}$ This is the basic equation which describes the relationship between the three fundamental pressures in a free stream **Eq. 1.1**

Eq. 1.1 can then be rearranged to the classic pitot measurement form (pitot being a device which determines the free stream velocity of the fluid) which is:

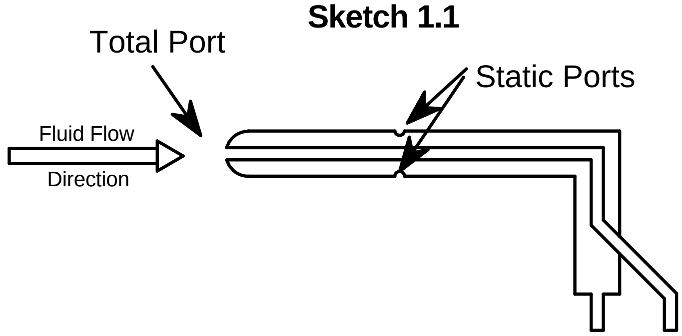
$P_{Dynamic} := P_{Total} - P_{Static}$ **Eq. 1.2**

Sketch 1.1 below shows a typical single point pitot-static tube for implementing *Eq. 1.2* to measure a single point velocity in a fluid stream. Note the Total and Static ports. The Total port is shown at the classic stagnation point. The pressure at the stagnation point is given as:

$P_{Stagnation} = P_{Total\_port} = P_{Dynamic} + P_{Sta}$

Which simply says that both the Stati
Dynamic Pressure exist at the Total I

Sketch 1.1



Also, at the Static port location on the sketch, the free stream velocity on th surface is zero due to the boundary layer effect, because stream flow is parallel to the side of the pitot and $P_{Dynamic}$=0, so:

$P_{Static\_port} := P_{Dynamic} + P_{Static}$

but, since $P_{Dynamic} = 0$ at the Static Ports, then:

$P_{Static\_port} := P_{Static}$

If the Static Port Pressure is subtracted from the Total Port pressure, then:

$P_{Dynamic} = P_{Total\_port}(P_{Stagnation}) - P_{Static\_port}$ (hence the name pitot-static: the total port with the stagnation pressure is normally called the pitot) **Eq. 1.2a**

How does this "reference pitot" perform based on Yaw angle? Data taken from **Ref. 1**:Ower, E. & Pankhurst, R. C. The Measurement of Air Flow, 5th ed., Pergamon Press, Fig. 3.22 is the "normalized value of the output reference to 0° for an Ellipsoidal nose pitot. The values at + and -30° were extrapolated from the lower points. The reference pitot values was assumed to be symetrical about the 0° axis

$k := 0..12$ range variable for the 13 data points taken

Yaw_angle :=

	0
0	-30
1	-25
2	-20
3	-15
4	-10
5	-5
6	0
7	5
8	10
9	15
10	20
11	25
12	30

Data 1.1

Reference_Ellip_pitot_static :=

	0
0	0.785
1	0.85
2	0.93
3	0.995
4	1.01
5	1.005
6	1
7	1.005
8	1.01
9	0.995
10	0.93
11	0.85
12	0.785

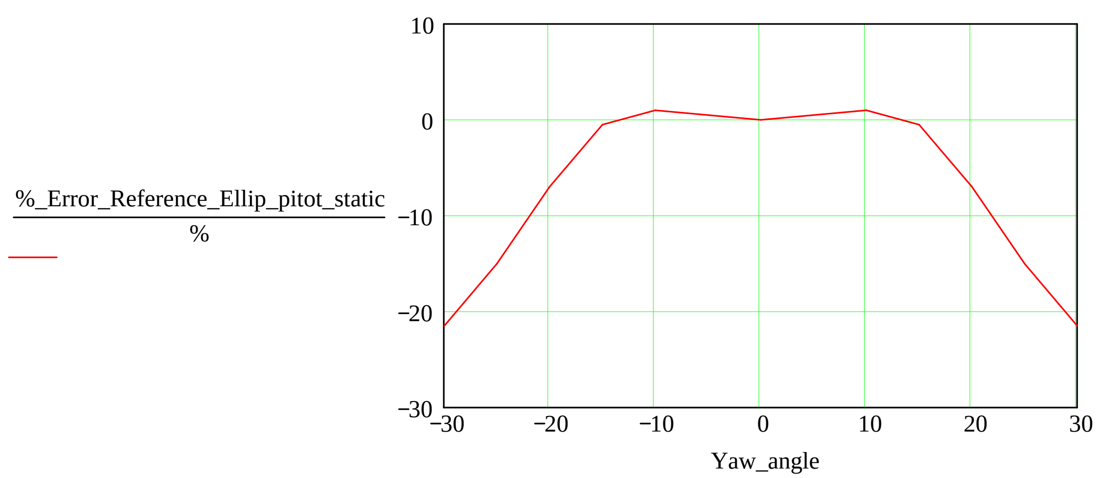
Data 1.2

To change this "normalized" data to % Error data of the reference pitot, Eq. 1.3 below is defined.

$$\%\_Error\_Reference\_Ellip\_pitot\_static_k := \frac{Reference\_Ellip\_pitot\_static_k - Reference\_Ellip\_pitot\_static_6}{Reference\_Ellip\_pitot\_static_6}$$

Eq. 1.3

GRAPH 1.1



Graph 1.1 shows that the reference Ellipsoidal nose pitot-static is affected very little by Yaw angle until the angle passes 15°. One would assume that alignment with the air stream would be fairly easy with this device.

2. Pressure Distribution Around a Cylinder

The pressure distribution around a cylinder in a flowing stream is well known. The equation for the pressure distribution is given by **Ref. 2**: Streeter, Victor L & Wylie, E. Benjamin Fluid Mechanics, 8th ed. McGraw-Hill, pg. 342 as:

$$P_{dynamic} := \frac{1}{2}\cdot\rho V^2\cdot\left(1 - 4\sin(\theta)^2\right)$$

Where: $P_{dynamic}$ is the dynamic pressure generated by the flowing stream, V and ρ are the velocity and density respectively of the flowing stream, and θ, the angle around the cylinder with 0° being the maximum pressure at the stagnation point. **Eq. 2.1**

For a visual representation of that pressure around a cylinder in a flowing stream , we can plot *Eq. 2.1* for ± 30°

if we set: $$y := \frac{Pdynamic}{\left(\frac{1}{2}\cdot\rho V^2\right)}$$

This is a technique which allows us to "normalize" or relate to the dynamic pressure without having to solve for it. When Pdynamic = ½ ρV^2, then y=1 **Eq. 2.2**

$j := 0 .. 60 \qquad x_j := j\cdot deg - 30\cdot deg$

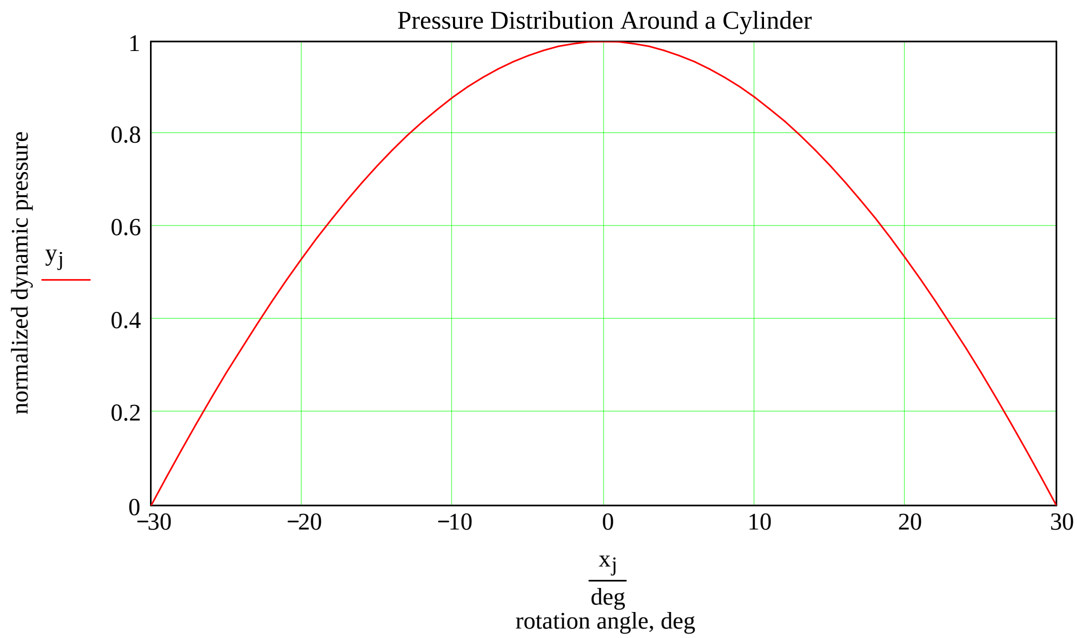
mathematically saying we will plot over a 60° range, starting at -30°

By substituting *Eq.2.1* into *Eq.2.2*, and expressing in terms of y, we get:

$$y_j := 1 - 4\cdot\sin(x_j)^2$$

the subscript **j** is a range variable as defined above **Eq. 2.3**

GRAPH 2.1



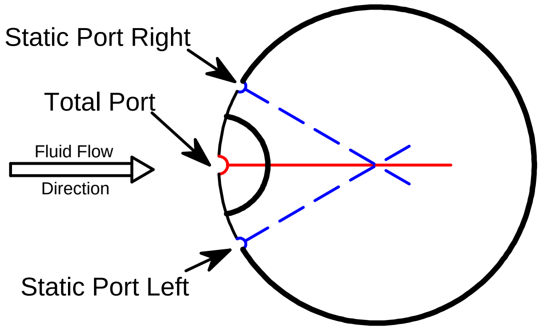
The plot shows that the pressure at 0° is = to 1, which is the dynamic pressure and at 30°, the pressure = 0

3. Using a Cylinder as a Pitot

To make a pitot-static measurement, we need two points in space which the stagnation pressure and static pressure exist. From Graph 2.1 above, the stagnation pressure (normalized dynamic pressure = 1) at 0° and the theoretical static pressure (normalized dynamic pressure = 0) exist at ±30°

If we make a pitot from a tube as indicated in Sketch 3.1 on the right, then according to *Graph 2.1*, the labeled required ports are shown.

Sketch 3.1



Another way to make the device which would aid in manufacturing, would be to use two tubes, one for the total port and one for the static ports, negating the need for a sealed tube within a tube as shown in *Sketch 3.1*.

From here, we should do a bit of analysis of the cylindrical pitot to determine if one should even bother to make the more complex and standard pitot of *Sketch 1.1*.

3.1 Theoretical Calculations

First, we can analyze the sensitivity to alignment of the static ports based on *Eq. 2.3*. The static error will be stated in terms of Dynamic Pressure since this is the desired result.

$$\text{Sensitivty\_Error\_\%} := \left(1 - \sin(31.5\cdot\text{deg})^2\right) - \left(1 - \sin(33.5\cdot\text{deg})^2\right) \qquad \textbf{Eq. 3.1}$$

$$\text{Sensitivty\_Error\_\%} = 3.163\%$$

missing the angular location of the static port by ±1° will result in a 3% dynamic pressure measurement error

Note: the actual location of the "static pressure ports" is Reynolds number dependent, and is usually located somewhere between 30 and 35° off total port centerline as indicated in *Sketch 3.1*. **Ref. 1**:Ower, E. & Pankhurst, R. C. The Measurement of Air Flow, 5th ed., Pergamon Press, pg. 37-38

The next question is how sensitive the static ports would be to yaw angle, where yaw would be defined as rotation about the centerline of the cylindrical tube.

We will assume that the two static ports are common to the central region of the tube as indicated and if the two ports are identical in diameter and depth, then it can be shown that:

$$P_{\text{Static\_port\_common}} := \frac{\left(P_{\text{Static\_port\_left}} + P_{\text{Static\_port\_right}}\right)}{2} \qquad \textbf{Eq. 3.2}$$

$$i := 0..12$$

range variable, from a value of 0 to 12

$$\text{left\_port\_angle}_i := i\cdot 5\cdot\text{deg} - 66\cdot\text{deg} \qquad \textbf{Eq. 3.3}$$

this statement causes the left_port_angle to go from -66° to +6° in five degree increments

$$\text{right\_port\_angle}_i := \text{left\_port\_angle}_i + 72\text{deg} \qquad \textbf{Eq. 3.4}$$

this statement causes the right_port_angle to lead the left_port_angle by 72° (36° around from total)

Note: The choice of 36° was chosen because it is the measured value of the example Cylindrical Pitot-Fechheimer Static tested in Section 4.

$rotation\_angle_i := \dfrac{right\_port\_angle_i + left\_port\_angle_i}{2}$	this equation defines the location of the Total port and is also the rotation angle or Yaw angle, with 0° aligned with the direction of flow	**Eq. 3.5**
$P_{Static\_port\_left_i} := 1 - 4\sin\left(left\_port\_angle_i\right)^2$	this is from *Eq. 2.3* above, defining the pressure in the left static port	**Eq. 3.6**
$P_{Static\_port\_right_i} := 1 - 4\sin\left(right\_port\_angle_i\right)^2$	same as above, except for the right port	**Eq. 3.7**
$P_{Static\_port\_common} := \dfrac{\left(P_{Static\_port\_left} + P_{Static\_port\_right}\right)}{2}$	all the variables are now defined for *Eq. 3.2*	

$$P_{Total\_port_i} := 1 - 4\cdot\sin\left(rotation\_angle_i\right)^2$$

this is the pressure at the Total Port as shown in the previous sketches and described theoretically as *Eq. 1.3*.

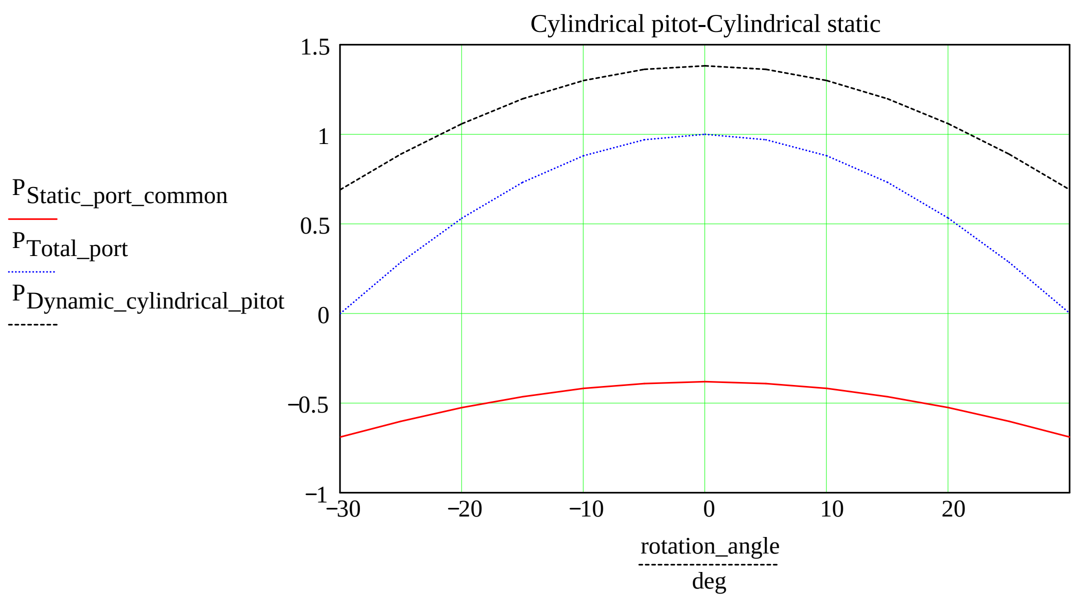
Eq. 1.2a can now be applied to determine the dynamic pressure from the cylindrical pitot - cylindrical static as a function of rotation angle

$$P_{Dynamic\_cylindrical\_pitot_i} := P_{Total\_port_i} - P_{Static\_port\_common_i}$$

from *Eq. 1.2a*

Graph 3.1 below is a plot of the two components, total and static, and the subsequent result--the Dynamic pressure

Graph 3.1

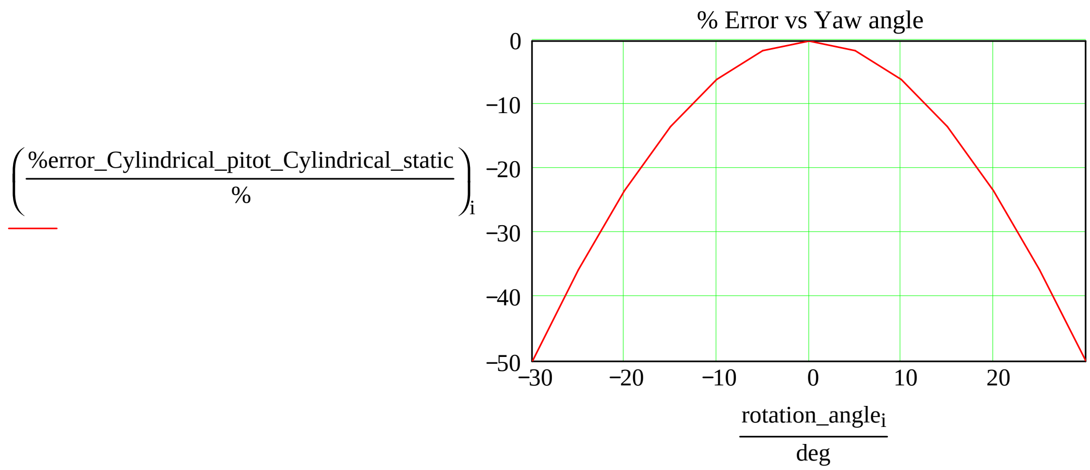


The error due to yaw or rotation angle can now be defined as:

$$\%error\_Cylindrical\_pitot\_Cylindrical\_static_i := \frac{P_{Dynamic\_cylindrical\_pitot_i} - P_{Dynamic\_cylindrical\_pitot_6}}{P_{Dynamic\_cylindrical\_pitot_6}} \qquad \textbf{Eq. 3.8}$$

Note: The subscript "6" is the data point for 0°
Graph 3.2 is a plot of Eq. 3.8 over the indicated Yaw angle

Graph 3.2



3.2 Observations from the theoretical model

1. The use of the cylindrical "static" ports causes extreme sensitivity of the calibration factor to the angular location of the "static" ports in manufacturing. To maintain 1% repeat between probes, this angle would have to be maintained within 0.3° (See *Eq. 3.1*). This type of cylindrical "static" port is sometimes called "Fechheimer" static.

2. Turbulent conditions and Reynolds number variation cause the static pressure location to vary (which almost always exist in a duct) **Ref. 1** :Ower, E. & Pankhurst, R. C. The Measurement of Air Flow, 5th ed., Pergamon Press, pg. 38.

3. The extreme sensitivity to Yaw angle of the Cylindrical pitot-cylindrical static makes it almost impossible to align instrument with flow direction. Most pitot-static devices are usually no more than ±1% error for ±10° yaw (see *Graph 1.1*). The Cylindrical pitot - Cylindrical static has ~10x this error in the theoretical model (See Graph 3.2). Since the "Fechheimer static" port is so sensitive to Yaw, it would make a good device to determine flow direction. One would need only to connect the two "Fechheimer statics" to some type of differential meter and adjust for null, at which time the point on the cylinder between the two would point in the flow direction. Port separation less than ±30° would reduce the sensitivity while ±30° to ±40° would be about the same sensitivity.

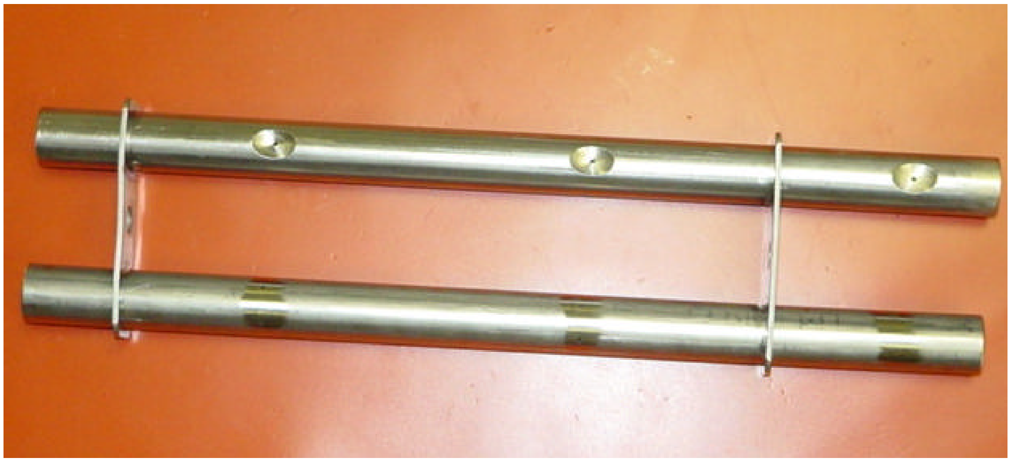
4. Since we have chosen an angle greater than 30° around from the stagnation point as the static pressure point, note from Graph 3.1 that the Dynamic pressure at 0° is more than 1. This means that the calibration factor of the pitot has been changed by the choice of the location angle of the "Fechheimer" static ports.

5. One should pick, in my opinion, a design for static pressure determination which would minimize Reynolds, turbulence, and surface contamination effects, not the calibration value since it is to easy to change a "factor" with little effect on accuracy.

4. Test of An Actual Example of a Cylindrical Pitot-Fechheimer Static Pitot

A picture of the Cylindrical pitot-Fechheimer static provided for testing is shown below as Fig. 1

Fig. 4.1



Note that there are two cylindrical tubes, with the top one containing the Total port and the bottom tube containing the Fechheimer static ports. The Fechheimer static ports are hard to see in this photograph due to their small size, but are in the sections of the bottom tube which appear to be polished. Also, note that there are three sets of totals and static ports along the length of the tubes. Additionally note that the Total ports are modified version of Sketch 3.1 and are "dished".

4.1 Measurements

Fstatic_port_diameter := 0.046·in the measurement of the static ports to ±0.002 in

Fstatic_port_angle := 36·deg the measurement of the static port location angle to ± 2 deg

Fstatic_cylinder_diameter := 0.76·in the measurement of the Fstatic cylinder diameter to ±0.01 in

Fstatic_cylinder_wall := 0.062·in the measurement of the Fstatic cylinder wall thickness to ±0.01 in

Cylindrical_dished_port_diameter := 0.0625·in the measurement of the total ports to ±0.002 in

Total_cylinder_diameter := 0.760·in the measurement of the total cylinder diameter to ±0.01 in

Total_cylinder_wall := 0.090·in the measurement of the total cylinder wall thickness to ±0.01 in

Total_port_dished_depth := 0.075·in depth of the ball end mill cut into the top of the total port to ±0.002 in

Total_port_radial_angle := 30·deg the the radial angle that the dished port covers around the total port

port_to_port_distance := 4.0·in the measure distance between the 3 sets of ports

Fcylinder_to_Ccylinder_spacing := 2.0·in how far the tube are apart to ± 0.1 in

tube_length := 12·in approximate length of the tubes

4.2 Procedure

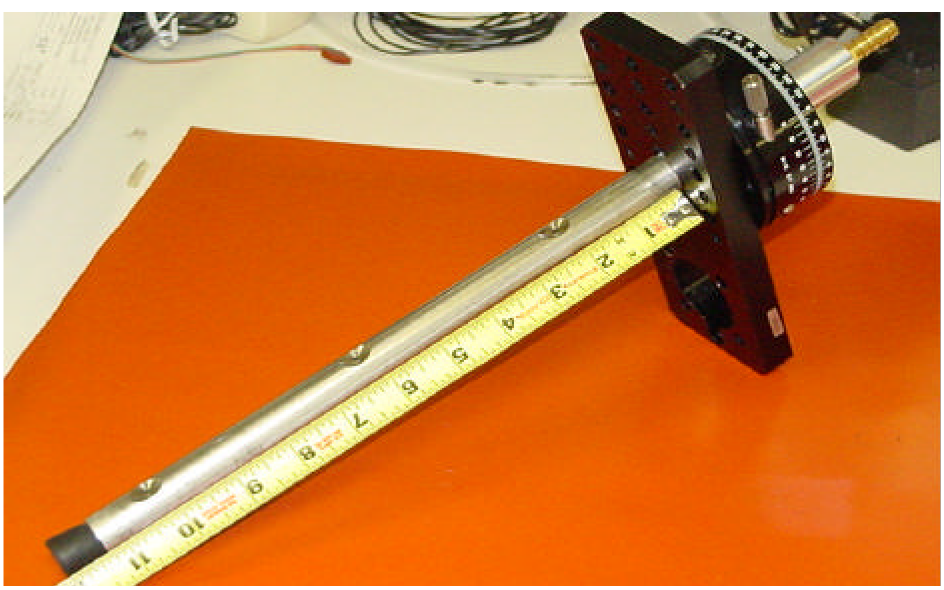
Since the individual probes (Total and Static) are side by side, and do not run in the "wake" of each other, it was determined that a more accurate way of testing for yaw angle sensitivity would be to dissemble the probe and test as a single unit. This way, as the probe was rotated, the ports would stay in nearly the exact same velocity stream.

The units were dissembled and then installed with the test fitting as shown in Fig. 4.2 for the Cylindrical Total.

Fig. 4.2

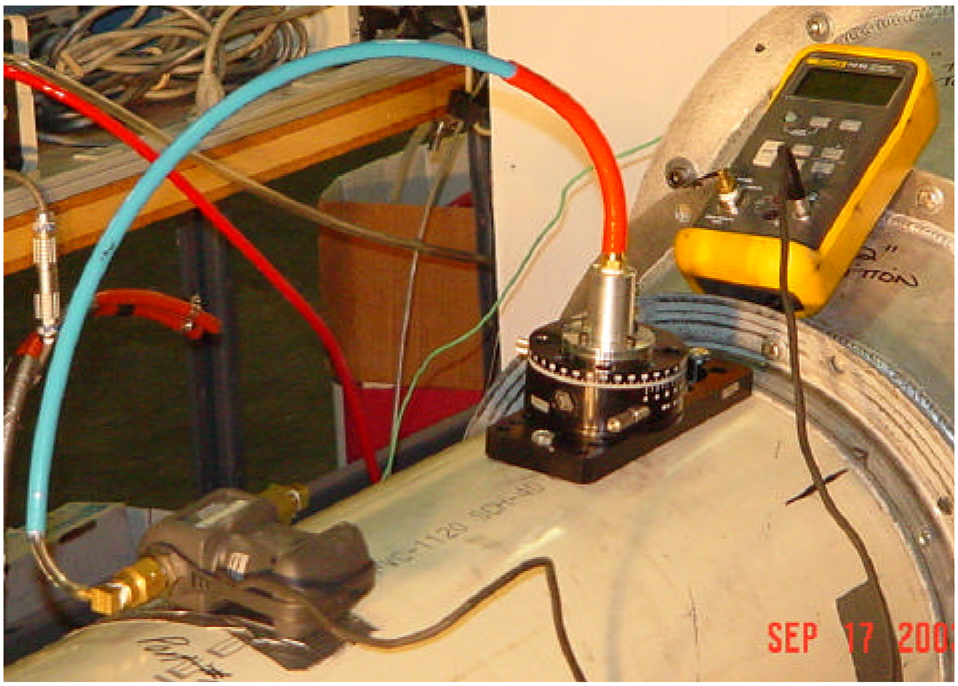
Fig. 4.3 shows the Cylindrical Total with the test fitting installed in the test fixture. Note that the ports are located 2 inches away from the wall of a 12 inch i.d. test duct to meet the requirement of 2½ diameters from the closed end of the cylinder as reccomended by **Ref. 1**, pg38.

Fig. 4 3



The test fixture was then inserted into a 12 inch schedule 40 plastic pipe ½ diameter from the transition which would provide a "flat" profile or velocity distribution so that all three port sets would be in approximately the same air velocity. An ANSI 210-85, Fig. 15 test set up was used to supply a known air flow to the 22 in to 12 inch transition shown. The air flow was then set to give an average velocity c 4000 ft/min in the 12 inch duct. The data was taken using a Fluke 718 30G pressure tester with a 10 inch water column remote head as shown.

Fig. 4.4



4.3 Data

Data was taken at 4000 ft/min duct velocity as supplied by the test chamber. The data was taken at each of the Yaw angles indicated for the Cylindrical total. The Fechheimer static probe was installed and the data was taken for it. The results are shown below, with Yaw angle in degrees and pressures in inches water column.

4000 ft/min was chosen because it is both a velocity range often encountered and also 1.0 in our measurement units of inches water column is the theoretical solution to $\frac{1}{2}\rho V^2$. This conveniently makes our actual test units and normalized units in the theoretical model very nearly the same.

Yaw_angle :=

	0
0	-30
1	-25
2	-20
3	-15
4	-10
5	-5
6	0
7	5
8	10
9	15
10	20
11	25
12	30

Data 4.

C_total :=

	0
0	1.057
1	1.127
2	1.21
3	1.293
4	1.354
5	1.39
6	1.405
7	1.4
8	1.373
9	1.33
10	1.26
11	1.19
12	1.11

Data

F_static :=

	0
0	0.563
1	0.503
2	0.467
3	0.417
4	0.383
5	0.336
6	0.322
7	0.305
8	0.291
9	0.269
10	0.259
11	0.235
12	0.24

Data 4.3

The F_static data was difficult to take due to excessive flucuations even in very smooth flow conditions This was also stated in **Ref. 1** , pg. 38

The Dished cylindrical total, however, was smooth and easy to determine.

4.4 Calculations

The Cylindrical Total-Fechheimer Static differential pressure must be calculated next. Since each was reference to atmospheric, then:

$k := 0 .. 12$ range variable for the 13 data points taken

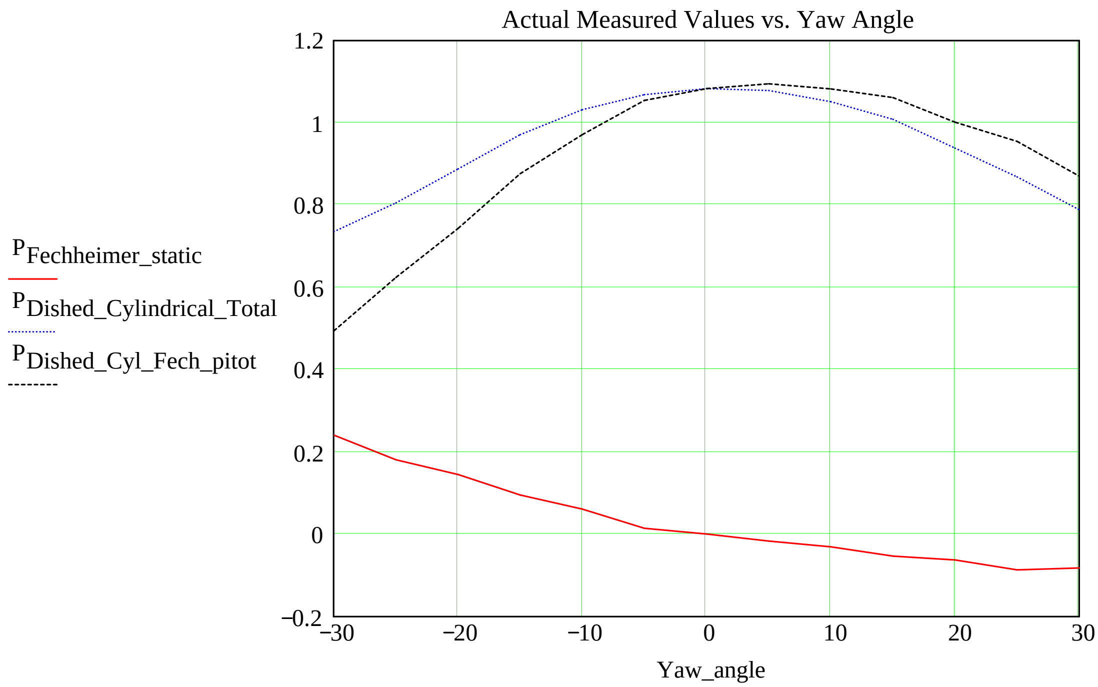
$P_{Fechheimer\_static_k} := F\_static_k - F\_static_6$ Since no separate measurement of duct pressure was made, the assumption is that the probe measures duct static at 0° **Eq 4.1**

$P_{Dished\_Cylindrical\_Total_k} := C\_total_k - F\_static_6$ Same reason as *Eq. 4.1* **Eq 4.2**

$P_{Dished\_Cyl\_Fech\_pitot_k} := C\_total_k - F\_static_k$ Essentially *Eq. 1.2* **Eq 4.3**

Graph 4.1 is a plot of

Graph 4.1



1. Compare this graph to **Graph 3.1.** Note that they are similar except for the static pressure from approximately +.25 to -.05 in a very non symmetrical manner. We can now determine how much error. From the theoretical calculations in Eq. 3.2, we can find the max and minimum values of first the theoretical Fechheimer absolute error:

$$\text{Absolute\_error\_theoretical\_Fstatic} := \max\left(P_{\text{Static\_port\_common}}\right) - \min\left(P_{\text{Static\_port\_common}}\right)$$ **Eq. 4.4**

$\text{Absolute\_error\_theoretical\_Fstatic} = 0.309$ Or the theoretical error rotating the port through ±30°

This can be repeated for the actual measurement Fechheimer Static by

$$\text{Absolute\_error\_measured\_Fstatic} := \max\left(P_{\text{Fechheimer\_static}}\right) - \min\left(P_{\text{Fechheimer\_static}}\right)$$ **Eq. 4.5**

$\text{Absolute\_error\_measured\_Fstatic} = 0.328$ note that our total differential is ~1 -- this is 32.8% error based on the dynamic or velocity pressure

This is good agreement between the theoretical and actual values.

Why is the error of the Fechheimer statics not symmetrical around 0°? For Eq. 3.2 (the pressure averaging equation) to hold, the diameter of the left and right Fstatic ports would have to be identical. It can be shown that the pressure averaging error is proportional to the fourth power of the ratio or

$$P_{\text{Fechheimer\_static}} := \frac{\left(\frac{\text{diameter\_left\_Fstatic\_port}}{\text{diameter\_right\_Fstatic\_port}}\right)^4 \cdot P_{\text{Fstatic\_left}} + P_{\text{Fstatic\_right}}}{\left(\frac{\text{diameter\_left\_Fstatic\_port}}{\text{diameter\_right\_Fstatic\_port}}\right)^4 + 1}$$ **Eq. 4.6**

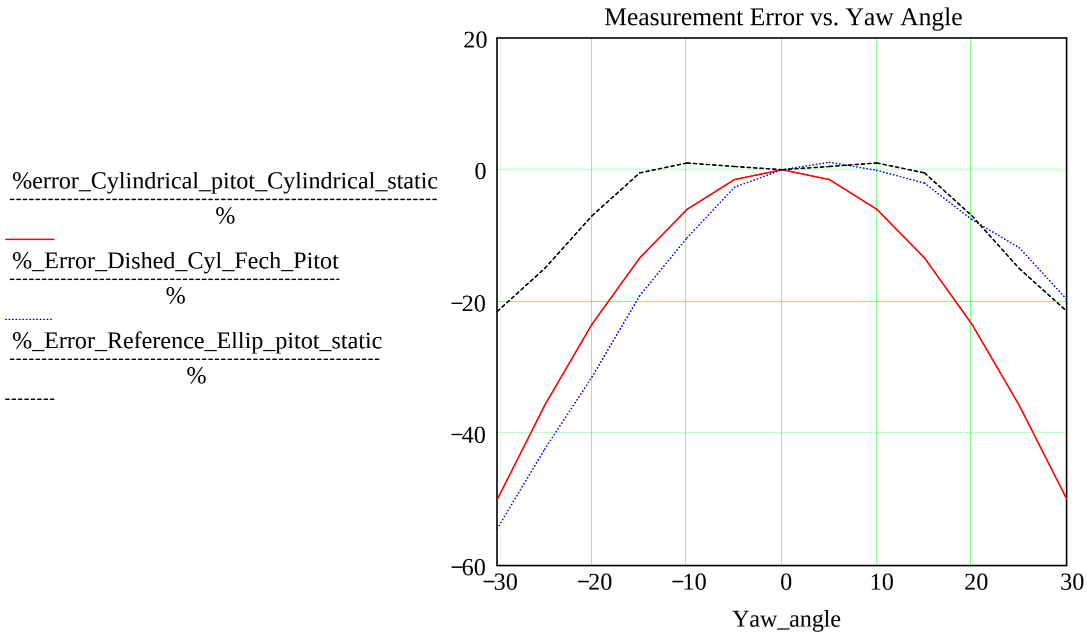
Based on the nominal diameter of 0.046 inches for the measured diameter Fstatic port, and error of ±0.001 inch would result in a ~15% pressure offset--very nearly what we are observing. This is a very difficult manufacturing issue, but even more important, and operational issue as the ports slightly plug. **2.** It should also be noted that even if the port diameters were identical, the error would still be the same, just centered around 0°.

3. We can now compare the actual Dished Cylindrical Pitot-Fechheimer Static in terms of % dynamic error to the theoretical analysis of a Cylindrical pitot and our reference Ellipsoidal nose pitot-static instrument. The dynamic or velocity pressure is the actual value used to determine flow in a duct.

$$\%\_Error\_Dished\_Cyl\_Fech\_Pitot := \frac{P_{Dished\_Cyl\_Fech\_pitot} - P_{Dished\_Cyl\_Fech\_pitot_6}}{P_{Dished\_Cyl\_Fech\_pitot_6}} \qquad \textbf{Eq. 4.7}$$

Note: In Graph 4.2 below, the Theoretical Cylindrical pitot-static is a solid red line
the actual measured Dished Cyl pitot-Fechheimer Static is a dotted blue line
the Reference ellipsoidal pitot -static device is the dashed black line

Graph 4.2



The theoretical cylindrical pitot and the actual measured dished cylindrical Fechheimer static compare fairly close --the solid red and the dotted blue. The measured value is not symmetrical about the zero axis because of the Fechheimer static offset error. (Item 1 above).

The referenced pitot-static device works well to a ±15° Yaw, whereas both the cylindrical pitots perform poorly on either side of 0°. One would find it difficult to both align the cylindrical type pitot to the air stream and manufacture it to any degree of repeatability.

4. This theoretical analysis and actual measurement of a cylindrical pitot-static supports the conclusion in **Ref. 1**:Ower, E. & Pankhurst, R. C. The Measurement of Air Flow, 5th ed., Pergamon Press, pg, 38, last paragraph "the pitot cylinder cannot be used for accurate determinations of flowrates in pipes..." He does conclude at the end of the section on pg. 39 that "the convenience of the pitot cylinder for local exploration of the flow will favor its use for many purposes where high accuracy is not essential. ...provided that the static pressure is determined by some other device."

5. The dished total seemed to perform slightly better than just the cylinder total (compare Graph 3.1 to Graph 4.1). However the overall effect of this on the dynamic pressure is minimal. To correct for total error, the static error must go in the opposite direction by an equal amount to make dynamic insensitive to yaw. By looking at Graph 4.1 PFechheimer_static, one may be tempted to say that could be accomplished. However, it is really an effect of diameter ratio error (Eq. 4.6) which just shifts error asymmetrical about the 0° axis.